Fitting Flats to Flats

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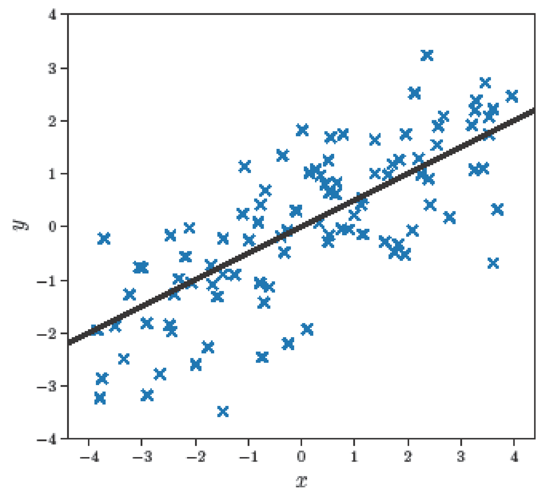
Motivation

- Fitting an affine subspace (flat) to a set of points is a standard task in graphics, vision, and engineering in general.



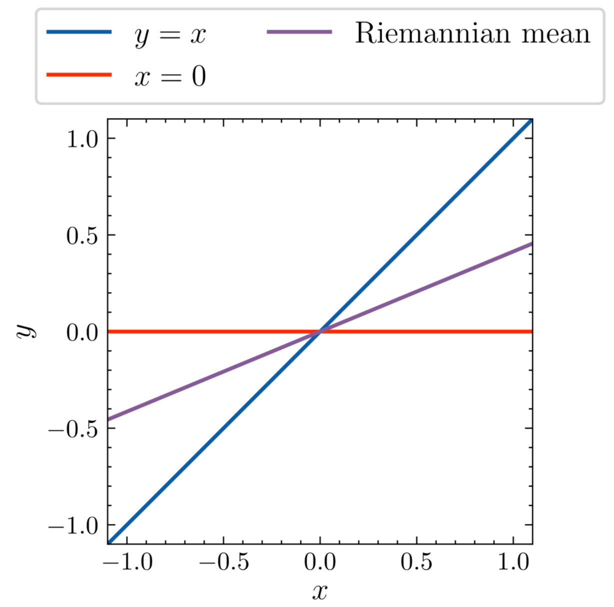
Typically solved by PCA or other Regression techniques.

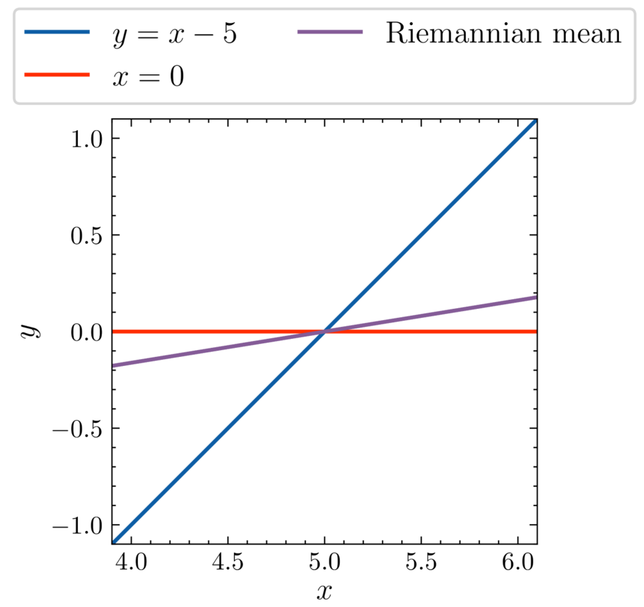
- No standard method to fit a flat to a set of other flats (of potentially varying dimensions).
- Reconstruction of line-like objects from multiple views → Fitting a line to a set of planes.
- Can be formulated as the Riemannian center on the Grassmannian of affine subspaces.

$$m = \underset{\mathcal{F} \in \mathrm{Graff}(k,d)}{\arg\min} \sum_i d^2(\mathcal{F}, \mathcal{F}_i)$$

- Problems:

No translation equivariance, difficult to understand





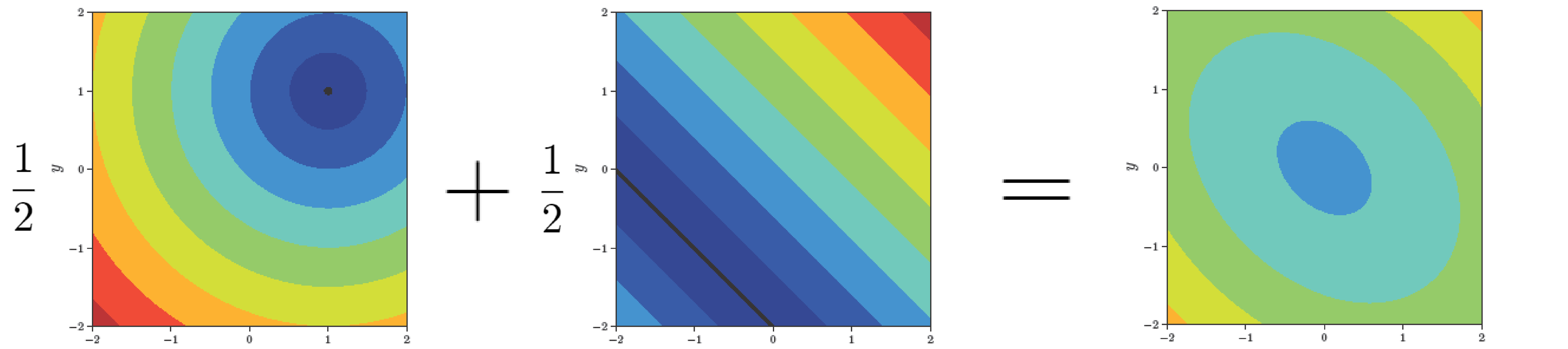
Method

Squared Distance Fields ...

- can be uniquely represented with a matrix-vector pair.

$$\mathbf{Q} \in \mathbb{R}^{d \times d}, \mathbf{r} \in \mathbb{R}^d$$

- can be linearly combined.



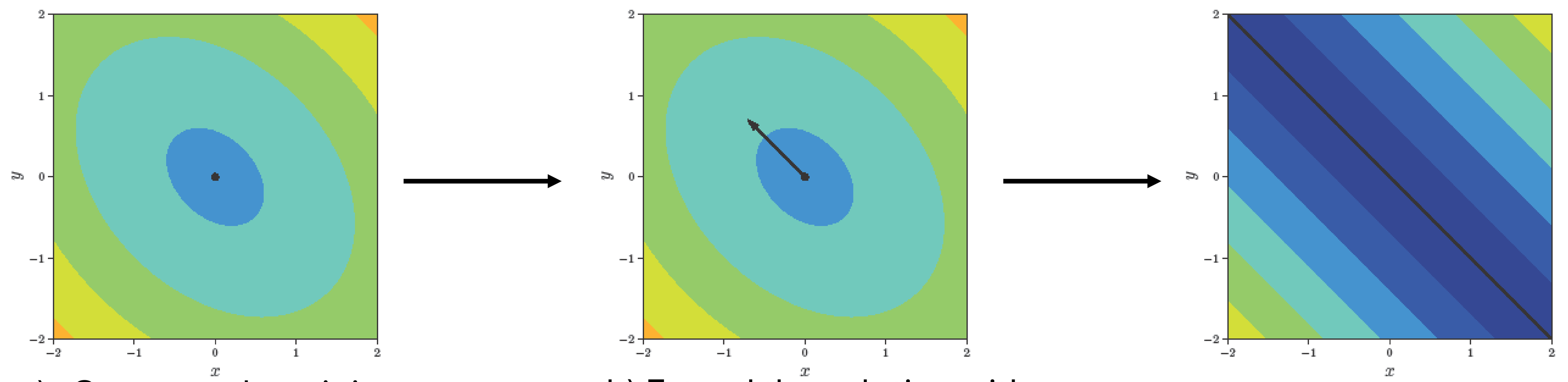
$$\frac{1}{2}(\mathbf{Q}_1, \mathbf{r}_1) + \frac{1}{2}(\mathbf{Q}_2, \mathbf{r}_2) = \left(\frac{1}{2}(\mathbf{Q}_1 + \mathbf{Q}_2), \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)\right)$$

Fitting a k-Flat to a set of other flats:

1. Compute any (weighted) L_p mean of the matrix-vector representation.

$$(\mathbf{Q}_1, \mathbf{r}_1), (\mathbf{Q}_2, \mathbf{r}_2), \ldots, (\mathbf{Q}_m, \mathbf{r}_m) \longrightarrow (\mathbf{Q}^*, \mathbf{r}^*)$$

2. Project the result onto the manifold of k-flats.



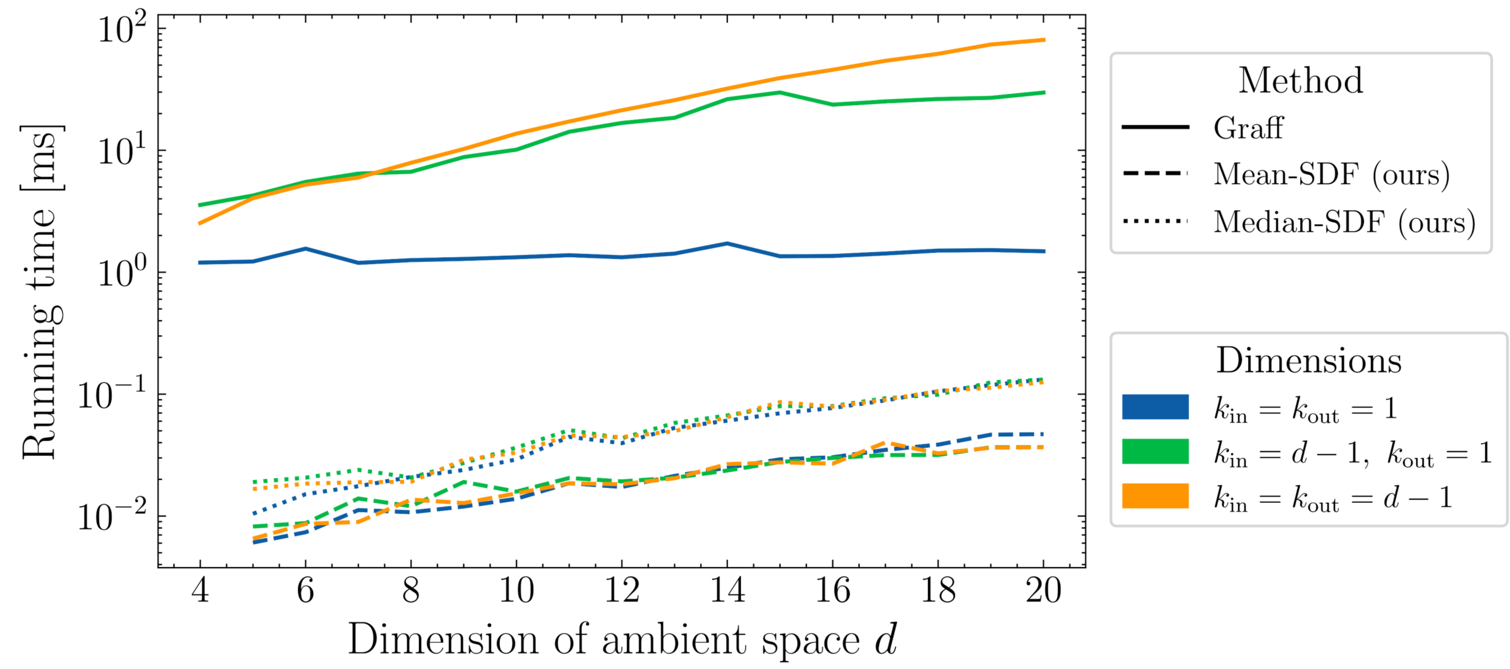
a) Compute the minimum of the quadratic function.

b) Extend the solution with k eigenvectors (smallest λ).

Resulting flat and SDF.

Results

- Equivariance w.r.t. all rigid transformations (including translations).
- Lower running time than methods on the Grassmannian of affine subspaces.



- Robustness to outliers.

Application: Multi-view reconstruction of line-like objects